28

Climate Modelling

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CONTENTS

28.1 Aim

This chapter aims to provide a statistically-oriented introduction to climate modelling. The following section starts by discussing the concept of climate and presents several different statistical interpretations. Section 28.3 then describes climate processes and modelling, and the main classes of climate model. Design of climate change experiments and ensemble simulation is covered in Section 28.4 followed by a discussion in Section 28.5 of how to make inference about actual climate from data produced by climate model simulations.

Selected references are given to relevant literature rather than attempting to provide a comprehensive review.

28.2 What is climate?

Climate is a statistical concept. Broadly speaking, climate is a statistical summary of weather, and hence climate science provides interesting challenges for environmental statisticians. Many of the issues in climate science rely on statistical insight e.g. the recent debate about the early-2000s slowdown (the so-called *hiatus*) in global warming (Fyfe et al. 2016).

A simple operational definition is that climate is a statistical summary of a sample of either observed or simulated weather data. For example, the 30 year time means of past weather observations from 1961-90 used by the World Meteorological Organisation to summarise *normal* conditions at different locations on the planet[1]. To account for seasonal variations, long-term means are generally computed separately for different seasons e.g. December-February mean for boreal winter and July-August for boreal summer. Such measures provide simple estimates of the typical weather one might expect to observe in different seasons at specific locations. However, this approach is rather limited for summarising weather that is not independent and identically distributed in time due to the presence of trends (Hawkins and Sutton, 2016) and modes of climate variability such as the El Ninõ-Southern Oscillation.

A more powerful interpretation of climate is to consider it to be an *expectation* of weather rather than a sample mean[2]. Expectation here should be understood not simply as the expectation of a weather variable but the expectation of any function of such a variable, for example, the exceedance of the variable above a high threshold that can be used to estimate the probability of extreme events (see Cooley and Sain 2010). Interpretation in terms of expectation relies upon representing weather probabilistically as a random variable. For example, time series models can be developed to represent historical data (see Section 3 of Chapter [crossref Craigmile and Guttorp chapter]) and then expectations can be calculated from such models. So rather than arbitrary sample statistics, climate may be considered to be the set of parameters in a probability model capable of faithfully representing weather data. At a deeper level, one might also consider these parameters to be uncertain and so climate is then a latent process for representing weather. In this sense, climate is a model concept rather than something that truly exists and can be found in the real world.

One may wonder where the uncertainty comes from for representing weather as a random variable: our incomplete unknowledge or fundamental indeterminism? A major source of aleatoric uncertainty arises due to the chaotic evolution of weather. Poincaré (1914) realised this when he wrote:

> *...even if it were the case that the natural laws had no longer any secret for us, we could still only know the initial situation approximately. If that enabled us to predict the succeeding situation with the same approximation, that is all we require, and we should say that the phenomenon had been predicted, that it is governed by laws. But it is not always so - it may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous*

[1] The original meaning of the word "climate" was "latitude zone of the Earth" but later evolved to mean the typical weather of such zones.

[2] "Climate is what we expect, weather is what we get" - Robert Heinlein in "Time Enough for Love" 1973

error in the latter…The meteorologists see very well that the equilibrium is unstable, that a cyclone will be formed somewhere, but exactly where they are not in a position to say; a tenth of a degree more or less at any given point, and the cyclone will burst here and not there, and extend its ravages over districts it would otherwise have spared…

Weather variables behave similarly to pseudo-random number generators in that there is rapid loss of information about initial conditions (i.e. the seed of pseudo-random number generators). This gives rise to what appears to be almost nondeterministic behaviour, which can then be described stochastically. Alternatively, one can define the probability of weather by considering ergodic invariant measures of attractors of dynamical systems (e.g. Drótos et al. 2015), however, the existence and uniqueness of such entities is not easy to prove especially for non-stationary systems.

At a more fundamental level, the evolution of weather is not strictly deterministic because of thermodynamic and quantum mechanical sources of uncertainty. Since weather is described by thermodynamic variables such as temperature, which are only statistical summaries of air molecules, it lacks information on the position and momenta of all the constituent air molecules. Furthermore, such variables can never be known with perfect certainty even for individual molecules due to quantum effects c.f. Heisenberg's uncertainty principle. The proverbial butterfly that creates chaos by flapping its wings (Lorenz, 1995), is in a similar quantum mixture of dead and alive states as Schrödinger's cat (Gribbin, 2011), and so can only perturb the atmosphere once it is observed to be alive. Climate models ignore such sources of indeterminism yet still simulate chaotic sequences of weather that are highly uncertain due to imperfect knowledge in the initial conditions and/or the model equations.

In summary, climate is a statistical concept that has several different interpretations. If one considers climate to be a probabilistic representation of weather then it is is a model-dependent concept defined by how we represent our beliefs. Climate is not so much an intrinsic part of physical reality but a particular and non-unique way of describing it. To paraphrase de Finetti's statement about probability, *"climate does not exist"* (De Finetti, 2017).

28.3 Climate modelling

Climate models are mathematical models for understanding and predicting the behaviour of the climate system (McGuffie and Henderson-Sellers, 2013). They represent our knowledge and beliefs about how climate processes operate.

28.3.1 Climate processes

The climate system is truly complex. However, physical conservation laws provide firm principles for model development. For example, one expects energy, mass, and momentum to be locally conserved, which is the basis for the dynamical equations used to model the atmosphere and oceans, as explained in the historical accounts of Roulston and Norbury (2013) and Edwards (2011).

The Earth's climate system is driven by solar radiation from the Sun. About one third of the solar radiation is reflected back out to space, and so 240 Wm^{-2} on average is absorbed primarily by the land and ocean surfaces. The Earth's surface then emits infra-red radiation back out to space. Gases in the atmosphere such as carbon dioxide, methane, water vapour etc. absorb infra-red radiation and so reduce the amount of outgoing infra-red radiation (*the greenhouse effect*). The tropics receive more solar radiation on average than the poles, which drives large atmospheric circulations, and in turn oceanic circulations, that both transport heat towards the high latitudes. Convective heating of the atmosphere and rotation of the Earth cause the atmospheric circulations to be fluid-dynamically unstable, which leads to the fascinating weather we experience on Earth.

To represent these processes realistically, it is necessary to model many coupled components that interact with one another in complicated ways. For example, the atmospheric winds drive the oceans, which in turn heat the atmosphere. Furthermore, these two fluid components interact with other components such as the cryosphere (e.g. sea ice) and the land surface including vegetation. In addition, processes on small spatial scales in the atmosphere and ocean are influenced by large-scale conditions but can also influence the larger scales e.g. storms can maintain contiental-scale high pressure systems.

28.3.2 Classes of climate model

Several different classes of climate model have been developed in the past 50 years. Climate models vary in dimensionality from simple conceptual models having only one variable (e.g. global mean temperature) to very high dimensional General Circulation Models having millions of variables required for resolving weather systems down to 10km horizontal resolution across the whole globe. The more sophisticated models explicitly represent atmospheric evolution and so can be thought of as pseudo-weather generators. The main classes of climate model commonly used in climate science are described below:

28.3.2.1 General Circulation Model (GCM)

GCMs (also sometimes referred to Global Climate Models) are augmented fluid dynamical models that aim to represent the general circulation of the global atmosphere and oceans. Atmospheric GCMs (AGCMs) evolved in the 1960s out of global Numerical Weather Prediction (NWP) models that were developed to make daily weather forecasts (Smagorinsky, 1983). The weather prediction models were extended to be able to do long climate simulations by adding simple representations of boundary components such as the land surface (e.g. crude hydrology models) and sea-ice. The development of AGCMs inspired the development of global ocean GCMs in the 1960s capable of describing the large-scale circulation in the world's oceans and then these were coupled to AGCMs to make so-called *coupled GCMs* (Manabe and Bryan, 1969). The fluid dynamics is modelled using a simplified form of the Navier-Stokes equations known as the *primitive equations* suitable for thin shells of fluid on a rotating sphere. This results in a coupled set of non-homogeneous 1st order partial differential equations having the general form:

$$\frac{\partial \mathbf{x}}{\partial t} = Q(\mathbf{x}, \nabla \mathbf{x}) + F(s,t) \tag{28.1}$$

where $\mathbf{x}(\mathbf{s}, \mathbf{t})$ is a vector field of prognostic variables defined over **s**pace-**t**ime, for example, temperature, horizontal wind velocity components, pressure, and humidity for the equations used to model the atmosphere in GCMs. The operator $Q(.)$ includes quadratic non-linearities caused by transport (advection) of mass, momentum, and energy (e.g. $du/dt = \partial_t u + u\partial_x u + v\partial_y u + w\partial_z u$). The general circulation is also a non-homogeneous forced-dissipative system where $F(.)$ represents external forcing and dissipation e.g. heating from radiation and the Earth surface, surface drag, etc. The solutions of this set of equations cannot be obtained analytically and so the equations have to be solved numerically. Numerical solution requires a finite representation and so these equations are usually approximated by ordinary finite difference equations involving values defined on a regular space-time lattice[3]. Current AGCMs typically have horizontal grid spacing of about 10-100km, time steps of around 10 minutes, and about 10-50 vertical levels. Coupled GCMs now typically produce around 2-10 years of simulated data per day when run on massively parallel supercomputers.

28.3.2.2 Regional Climate Model (RCM)

Because of their coarse horizontal resolution, GCMs misrepresent smaller scale features (e.g the Alps), which are known to influence on local weather and climate. One way to try to overcome this problem is to *downscale* global GCM output by using it to force a higher resolution climate model that covers only a specific region. This nested approach can be particularly useful for investigating more extreme weather such as storms and extreme local rainfall e.g. RCM projections of extreme weather over Europe (Beniston et al., 2007). Although providing more spatial detail in the simulated weather, downscaling neglects how this may feed back onto larger spatial scales represented by the GCM. Regional climate models generally have similar fluid equations to those of GCMs but often have more detailed surface parameterisations e.g. orography, vegetation, wind gusts, etc. For more details of the added value of such an approach see Feser et al. (2011).

28.3.2.3 Earth System Model (ESM)

In recent decades, much attention has been paid to adding explicit representations of bigeochemical processes to coupled GCMs. For example, rather than prescribing carbon dioxide concentrations in the atmosphere by adjusting $F(s,t)$, many models now calculate carbon dioxide concentrations based on how the carbon cycle might respond to carbon dioxide emissions. This involves modelling carbon sources and sinks in the biosphere and increases the number of prognostic variables. Other biogeochemical processes that are now represented include aerosols and sulphur cycle and ozone chemistry. Earth System Models are increasingly used in climate change projections - see Flato (2011) for a review. For computational speed, fast conceptual versions of ESMs have also been developed known as Earth system Models of Intermediate Complexity (EMIC), which have highly simplified representations of the atmosphere (see Claussen et al. 2002).

28.3.2.4 Low-order models

In addition to models having many variables, it is also useful to consider highly simplified conceptual models of climate with only a few (or even one) variables. For example, a simple heat balance equation can provide much insight into how global mean temperature $x(t)$ will respond to future changes in radiative forcing $F(t)$ (Gregory and Forster, 2008)

$$C\frac{dx}{dt} = -\lambda x + F(t) \tag{28.2}$$

where C is the heat capacity of the atmosphere and Earth surface and λ is the strength of the feedback in the system. Such models representing averages of variables over space have been widely used to improve process understanding e.g. single vertical column radiative-convective models of the atmosphere, box models for oceanography and carbon cycle, etc.

[3] some AGCMs use a truncated set of spherical harmonics rather than finite differences to calculate horizontal spatial derivatives.

28.3.2.5 Stochastic Climate Models

The models above are approximations to a continuum because they do not represent variations smaller than a predefined spatial resolution. One can argue that such truncation could and should be partially remedied by adding stochastic noise to the forcing to account for unrepresented sub-grid processes (Palmer and Williams, 2008). For example, a simple yet widely used stochastic climate model can be constructed by adding Gaussian white noise to the right hand side of Eqn. (28.2) i.e. $F(t) \to F(t) + \epsilon$ where $\epsilon \sim N(0, \sigma^2)$ (Hasselmann, 1976; and references therein). The noise term represents irregular weather variability and the resulting stochastic differential equation can easily be solved by spectral or finite difference methods. The resulting *red noise* process is similar to a first order autoregressive AR(1) process. This approach can easily be extended to most other low order conceptual models (Wilks, 2008). Another motivation for adding randomness to the forcing is to account for our epistemic uncertainty in the physical parameterisations. It is hoped that by making a climate model inherently stochastic, one may be able to reduce and even eliminate biases present in the deterministic model. However, unlike the conservation principles used to formulate the deterministic component, there is very little guiding theory on how to correctly specify the random components.

In summary, in the past 50 years, climate modellers have developed a rich hierarchy of models ranging from low order models to high resolution models having thousands of grid point variables. The high resolution high complexity models are the main tools used to make climate change predictions supported by understanding gained from the simpler models.

28.4 Design of Experiments for Climate Change

Design of climate model experiments is limited by the huge computational expense of running GCMs and by the diverse range of different applications they attempt to inform. Up until the mid-1990s, it was common to publish analysis of single simulations from individual climate models, some of which did not even include an annual cycle in radiation e.g. so-called *perpetual January* simulations. With such an approach, inference about the real world was based on implausible what-if assumptions such as “what would the actual climate do if it behaved like that of this model”.

With increasing computing power, climate scientists are now able to explore a wider range of possibilities by running sets of simulations known as *ensembles* (Collins et al. 2012). Such ensemble data can be used to perform sensitivity analysis and uncertainty quantification. Various types of ensemble are used to sample the different sources of uncertainty. Ensemble experiments allow one to test the sensitivity of results to choice of initial conditions, model parameters for a given climate model, future emission scenarios, and choice of climate model. These types of ensemble are briefly described below - for more information see Flato et al. (2013).

Simulated data from such ensembles together with past observations provides the data from which one then attempts to infer the future behaviour of the single realization of the real climate system (Stephenson et al. 2012).

28.4.1 Initial condition ensembles (ICE)

Because of the chaotic nature of weather simulated by climate models, the time series produced by such models are highly sensitive to initial conditions i.e. after a short period of around two weeks, small differences in atmospheric initial conditions lead to very different realisations of simulated weather. This aleatoric uncertainty due to natural variation puts a fundamental limit on how accurately climate can be predicted and so needs to be quantified. Climate models are generally not ergodic, especially in the presence of forcings having long-run trends, and so it is necessary to do repeated simulations of climate models for each choice of model parameters. Such replication is computationally expensive and different modelling centres make different choices as to how perturb initial conditions (e.g. in the atmosphere and ocean components) and how many runs (replications) to make. With state-of-the-art climate models, typically from 1-10 simulations are made for each choice of model parameters (see Table 28.1).

28.4.2 Perturbed Physics Ensembles (PPE)

In addition to fundamental constants such as the radius of the Earth, climate models also have many tunable parameters that need to be prescribed. For example, the latest climate model used by the Met Office in the UK currently has around 50 tunable parameters in addition to parameters that can be estimated from measurements e.g. the spatial distribution of vegetation types, radiative absorption spectra of gases such as carbon dioxide, etc. (personal communication, David Sexton and Mat Collins). The simulated climate is sensitive in varying degrees to the choice of each these parameters and how they interact. Most of the parameters are usually chosen to be "best guess" values guided by past experience and scientific knowledge and then a small subset are adjusted (*tuned*) after running short simulations in order to minimise excessive biases in the simulations. Because of the high dimensionality of the parameter space and the computational expense of making simulations, it is not generally feasible to tune the parameters automatically unless one makes use of statistical emulators to interpolate the response behaviour in parameter space e.g. Williamson and Blaker (2014).

28.4.3 Multi-Model Ensembles (MME)

A further source of uncertainty arises from the structure of climate models being different. Climate modellers make choices about which equations to use and how to implement them numerically, which leads to models having different schemes and parameter sets. This gives rise to additional uncertainty not captured by PPEs (Yokohata et al., 2012).

Comparison of coupled GCMs has been greatly facilitated by Coupled Model Intercomparison Projects (CMIPs) organised by the World Climate Research Programme's Working Group on Coupled Modelling. The Lawrence Livermore National Laboratory collected simulations of the past, present and future climate in 2005-6 for phase 3 (CMIP3) and in 2011-13 for phase 5 (CMIP5). The vast amount of simulated data from more than 20 modelling centres is made freely available on online servers to allow analysis by scientists outside the modeling centres (36 terabytes for CMIP3 and 2 petabytes for CMIP5; personal communication, Karl Taylor). The resulting publications inform the climate assessment reports produced by the Intergovernmental Panel on Climate Change (IPCC). CMIP also provides an invaluable framework for coordinated design of common sets of experiments that modelling centres then perform with their models (Taylor et al. 2012).

TABLE 28.1
Climate change simulations made for the most recent phase 5 of the Coupled Model Intercomparison Project (http://cmip-pcmdi.llnl.gov/cmip5/). Subsets of the data can be visualised and downloaded from https://climexp.knmi.nl.

Modelling Centre	Country	Climate Model	Number of simulations				
			Historical	RCP2.6	RCP4.5	RCP6.0	RCP8.5
BCC	China	BCC-CSM1.1	3	1	1	1	0
		BCC-CSM1.1-m	3	1	1	1	0
CCCma	Canada	Can ESM2	5	5	5	0	5
CMCC	Italy	CMCC-CM	1	0	1	0	1
		CMCC-CMS	1	0	1	0	1
CNRM-CERFACS	France	CNRM-CM5	10	1	1	0	5
CSIRO-BOM	Australia	ACCESS1.0	1	0	1	0	1
		ACCESS1.3	3	0	1	0	1
CSIRO-QCCCE	Australia	CSIRO-Mk3.6.0	10	10	10	10	10
EC-EARTH	Europe	EC-EARTH	9	2	7	0	8
FIO	China	FIO-ESM	3	3	3	3	3
GCESS	China	BNU-ESM	1	1	1	0	1
INM	Russia	INM-CM4	6	4	4	1	4
IPSL	France	IPSL-CM5A-LR	6	4	4	1	4
		IPSL-CM5A-MR	3	1	1	1	1
		IPSL-CM5B-LR	1	0	1	0	1
LASG-CESS	China	FGOALS-g2	5	1	1	0	1
MIROC	Japan	MIROC5	5	3	3	3	3
		MIROC-ESM	3	1	1	1	1
		MIROC-ESM-CHEM	1	1	1	1	1
MOHC	UK	HadGEM2-AO	1	1	1	1	1
		HadGEM2-CC	1	0	1	0	1
		HadGEM2-ES	4	4	4	3	4
MPI-M	Germany	MPI-ESM-LR	3	3	3	0	3
		MPI-ESM-MR	3	1	3	0	1
MRI	Japan	MRI-CGCM3	3	1	1	1	1
NASA GISS	US	GISS-E2-H p2	5	1	5	1	1
		GISS-E2-H p3	6	1	5	1	1
		GISS-E2-H-CC p1	1	0	1	0	0
		GISS-E2-R p1	6	1	6	1	1
		GISS-E2-R p2	6	1	5	1	1
		GISS-E2-R p3	6	1	6	1	1
		GISS-E2-R-CC p1	1	0	1	0	0
NCAR	US	CCSM4	6	5	6	6	6
NCC	Norway	NorESM1-M	3	1	1	1	1
		NorESM1-ME	1	1	1	1	1
NIMR/KMA	Korea	HadGEM2-AO	1	1	1	1	1
NOAA GFDL	US	GFDL-CM3	5	1	1	1	1
		GFDL-ESM2G	3	1	1	1	1
		GFDL-ESM2M	1	1	1	1	1
NSF-DOE-NCAR	US	CESM1(BGC)	1	0	1	0	1
		CESM1(CAM5)	3	3	3	3	2

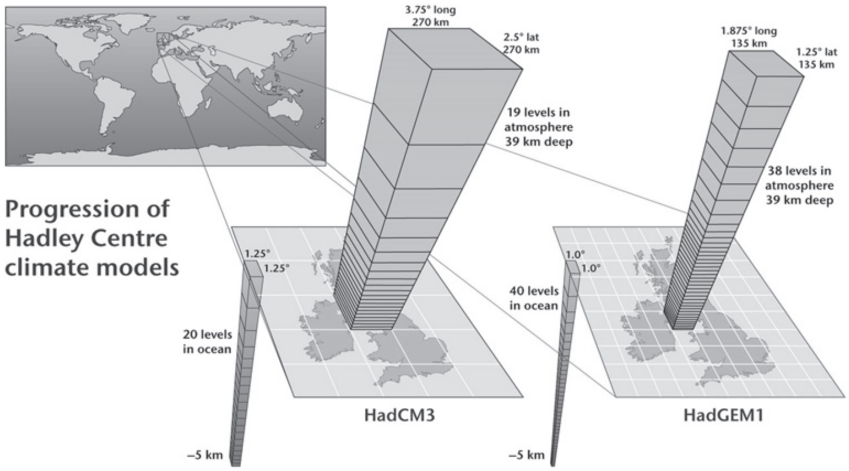

FIGURE 28.1
Schematic showing the grid cells used in recent GCMs developed by the UK Met Office. (Reprinted with permission ©Crown Copyright Met Office).

28.4.4 Climate change projections

Table 28.1 summarises the climate change simulations made with 42 models from 22 modelling centres for the most recent CMIP5 experiment. Climate simulations were made using greenhouse gas concentrations in the 20th century (historical) and in four scenarios known as Representative Concentration Pathways (RCPs), which represent how greenhouse gas emissions may evolve in the future. The four RCPs, RCP2.6, RCP4.5, RCP6.0, and RCP8.5, are named after equivalent radiative forcing values in the year 2100 relative to pre-industrial values (see Fig. 28.2). RCP2.6 optimistically assumes that global annual greenhouse gas emissions peak between 2010-2020, with emissions declining substantially thereafter, whereas RCP8.5 assumes that emissions will continue to increase throughout the 21st century. Emissions peak around 2040 and 2080 for intermediate scenarios RCP4.5 and RCP6.0, respectively.

Modelling centres decide how many different initial condition simulations to make for each scenario, which leads to an unbalanced design as can be clearly noted in Table 28.1. The design of such experiments is complex because of many competing scientific and policy needs (see CMIP6 rationale in Eyring et al. 2016). However, it is worth noting that such designs have so far made very little use of statistical knowledge about design of experiments in order to obtain better balanced experiments e.g. optimal size of ensembles, balance between different scenarios, etc.

Figure 28.3 summarises the global mean surface temperature response from these CMIP5 simulations under the different scenarios. The mean of all the simulations clearly reveals warming for all the scenarios, with RCP6.0 and RCP8.5 leading to more than 2^0C additional warming by the end of the 21st century. These results provide important evidence for informing policy makers on climate change strategies. For more discussion and interpretation of these results see Collins et al. (2013).

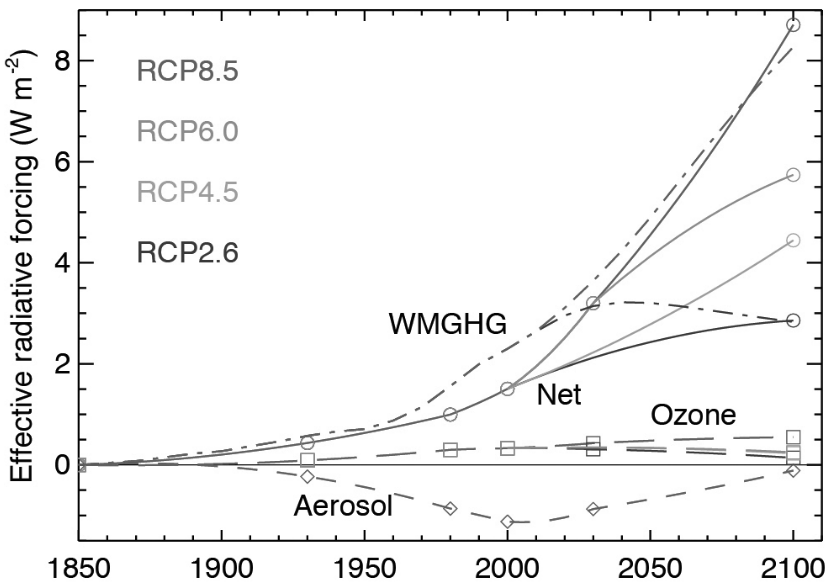

FIGURE 28.2
Effective global mean radiative forcing for the scenarios used in recent CMIP5 simulations: colours indicate the RCPs with red for RCP8.5, orange RCP6.0, light blue RCP4.5, anddark blue RCP2.6. (Reprinted with permission from Myhre et al. (2013)).

28.5 Real world inference from climate model data

It remains an important statistical challenge of how to best use climate model and observational data to infer the behaviour of the real climate system (Stephenson et al. 2012).

28.5.1 Current practice

The most common approach for estimating the expected climate change response is to calculate the multi-model mean of the MME i.e. the arithmetic mean of the individual model simulations e.g. Fig 31.3. This simple yet heuristic approach of one vote per model gives equal weight to each climate model irregardless of a) how many simulations each model has contributed, b) how interdependent the models are or c) how well each model has performed at simulating past climate (Sansom et al., 2013). No use is made of past observations unless past *performance metrics* have been used to screen out poorly performing models before taking the mean (e.g. Santer et al., 2009). This approach ignores model errors and provides only an estimate of the mean response with no credible estimate of how uncertain the individual response of the real climate system is likely to be.

28.5.2 Imperfect climate model and observational data

Despite models being able to capture many of the features of the climate system, climate scientists continue to spend much time striving to develop models that have smaller differ-

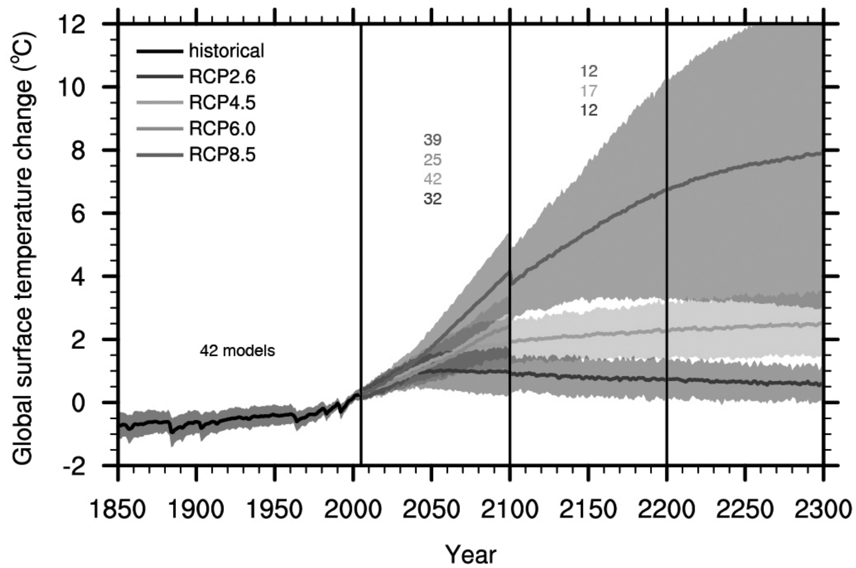

FIGURE 28.3
CMIP5 simulated global mean surface air temperature differences from 1986-2005 means for different RCP scenarios. The multi-model means are shown as solid lines for each RCP and ensemble spread is indicated by shading (1.64 standard deviation). Only one ensemble member is used from each model and numbers in the figure indicate the number of different models contributing to the different time periods. Discontinuities at 2100 are due to different numbers of models performing the extension runs beyond the 21st century and have no physical meaning. No ranges are given for the RCP6.0 projections beyond 2100 as only two models are available. (Reprinted with permission from Collins et al. (2013)).

ences to the observed behaviour of the climate system. Many of these biases are substantial compared to the climate change response and have resisted many years of model development e.g. double equatorial convergence zones, overly zonal storm tracks, too little monsoon rainfall over India, etc..

Rather than compare imperfect models with imperfect observations, it is useful to consider the concept of *model discrepancy* defined as the difference between the *actual climate* (i.e. that observable with perfect measurements) and climate model output. Model discrepancy can be considered to be analogous to *observation error* i.e. the difference between the actual climate and observations. In principle, by specifying statistical models for both model discrepancy and observation error, it is possible to then infer actual climate from simulated data from climate models and data from past observations.

However, unlike observation error, very little theory or guidance exists on how to specify model discrepancy. While model discrepancy is most likely independent of observation error, it is certainly not independent between different models due to the presence of common biases that are caused by various model components being related to one another (Knutti et al., 2013) Furthermore, model discrepancy is unlikely to be stationary in time and beliefs about how it may change have been shown to have substantial impact on predicted future climate (Buser et al. 2009; Ho et al. 2012).

28.5.3 Probabilistic inference

To go beyond heuristic predictions, it is necessary to make explicit assumptions about model discrepancies and observation error. The following sections briefly review various approaches that have been proposed for making statistical inference from multi-model ensembles.

28.5.3.1 The truth-centered approach

The simplest interpretation of an ensemble is the so-called *truth-centered* paradigm, which assumes that the model simulations are independent and identically distributed (i.i.d.) about the observed climate. Such a framework emerged naturally in Tebaldi et al. (2005) as a probabilistic interpretation of the heuristic *reliability ensemble averaging* method introduced by Giorgi and Mearns (2003). Multivariate extensions were then developed by Smith et al. (2009) and Tebaldi and Sansó (2009) and a related spatial model was proposed by Furrer et al. (2007). It should be noted that these frameworks neglected various sources of uncertainity such as observational uncertainty and natural variability in model simulations.

Common model discrepancy was also not represented until Tebaldi and Sansó (2009) proposed treating it as a fixed effect to be estimated. However, a more flexible approach is to treat common model discrepancy as a random effect. Chandler (2013) proposed a random effects framework where instead of being distributed around observed climate, model simulations are considered to be distributed about a latent variable (the ensemble consensus), which in turn is distributed about the actual climate.

28.5.3.2 The coexchangeable approach

Rather than make strong i.i.d. assumptions, Rougier et al. (2013) and Rougier and Goldstein (2014) used the concept of exchangeability to propose an alternative framework capable of representing common model discrepancy. The basic idea is to assume that a carefully chosen subset of model simulations are exchangeable with one another (i.e. statistically indistinguishable), and are also exchangeable with a linear transformation of the observations. Exchangeability is represented by introducing a random ensemble consensus variable, but in contrast to Chandler et al. (2013), the direction of conditioning is reversed - the actual climate is distributed about the ensemble consensus.

28.5.4 Summary

Despite progress in developing inferential frameworks, there is still much more to be done, such as how to prescribe discrepancy parameters, which are extremely difficult to elicit from climate scientists due to the lack of any guiding principles on common bias. The existing multi-model frameworks also neglect various sources of uncertainty, for example, how to make optimal use of perturbed physics ensembles to account for tuning of climate model parameters. The current frameworks also do not address *emergent constraints* i.e. the climate change response being dependent on the present day state of each model (Bracegirdle and Stephenson, 2012). Finally, the estimation methods are generally quite slow (e.g. MCMC) and so are not easy to apply to large spatial gridded climate data sets.

28.6 Concluding remarks

This chapter has provided an introduction to the concept of climate and how it is modelled. There is clearly a need for statistical modelling and interpretation in climate science. The very concept of climate is statistical yet is still not unambiguously defined. Better statistical interpretation could help avoid pitfalls associated with misleading absolute concepts such as "THE climate trend". Statisticians could also be usefully engaged in the development and testing of stochastic climate models, which require both physical and statistical expertise. Design of experiments and model tuning are other obvious areas that need more attention, for example, CMIP could produce more accurate projections if the number of simulations in past and future scenarios were more balanced. Finally, statisticians have a critical role to play in helping to make climate data relevant for decision-making, for example, in making reliable inference about future regional climate change.

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